The Bond Graph Method
applied to
Social and Life Sciences

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Abstract
Considering the success of Bond Graph technology in engineering sciences since its invention by Paynter in 1959, it is increasingly interesting to investigate its application to other fields. This paper attempts to relate Bond Graph technology to Social and Life Sciences, utilizing the Bond Graph structure as a basis for searching for equivalencies to the Energy concept inherent in Bond Graphs. The mathematical theory, which describes the behavior of entities in the social and biological fields, is presented in the context of it's relation to Bond Graphs. Considering that many phenomena in these fields are also represented by mathematical models that deal with partial differential equations, this research attempts to relate this modeling process to the application of the Bond Graph method.

1. INTRODUCTION

Since the end of the Second World War and specially since the introduction of computers, a lot of scientific effort has been put into the modelling and simulation of Dynamic Systems in the fields of Social Science (FORRESTER 61, FORRESTER 68, FORRESTER 71, FITZROY 76, COYLE 77, LILIEN 83, LILIEN 92) and Life Sciences (GOLD 77, JOLIVET 82, LEBRETON 82, BERTRANDIAS 90, CHERRUAULT 98).

Simultaneously, much work has been done in the Statistics field of Multivariate Data Analysis to construct efficient mathematical tools for understanding the behavior of social and biological entities (MORRISON 67, JOHNSON 82, KRZANOWSKI 88, LEBRAT 97).

Finally, powerful methods have been developed to model reality with Partial Differential Equations (FARLOW 82, BELTRAMI 87, FOWLER 97, BASSANINI 97) and to compute their solutions (MOHR 92, CURNIER 93, ZIENKIEWICZ 94, BELYTSCHKO 00).

The challenge is to integrate all the methods developed so far with the tools that have been developed in Automatic Control Theory as used in Engineering (FRANKLIN 94, DORF 95, LONGCHAMP 95). To do so, the equations used to describe the behavior of the systems must be related to an analytical or logical structure which should allow the use of concepts such as energy, stability, observability, controllability, frequency response and so on, to explain the system behavior.

Can Bond Graph representations be used systematically to understand the behavior of social and biological systems? If so, what are the underlying fundamental mathematical considerations and their relations to Bond Graphs? These questions are explored below.

2. MATHEMATICAL FUNDAMENTALS

Bond Graphs allow one to represent in a single language many scientific domains connected together (KARNOPP 00). When we consider what Paynter (PAYNTER 60, p. 32) did to establish the basis of Bond Graphs, we realize that he started with the Fundamental equation of energy continuity:

$$- \operatorname{div} \bar{p} = \frac{\partial \varepsilon}{\partial t} + p_d \qquad (1)$$

where $\bar{p}$ is the power flow, ε is the energy density and p_d is the energy dissipation.

This fundamental equation is grounded in the field of Differential Geometry (ABRAHAM 83, DODSON 91, FRANKEL 97, HUBBARD 99) and is used to describe the dynamics of a Continuous Medium in a space, whether physical or abstract.

This research explores basic principles which apply to continuous entities as stated in the continuous media theory by many authors (TRUESDELL 65, OTTINO 89, COIRIER 97, HJELMSTAD 97, FREY 98, MASE 99, WIELGOSZ 99) and shows their application to other fields as well as their relation to Bond Graphs.

Let us consider a control volume B with boundary R in a multidimensional-space x as shown in Fig. 1:

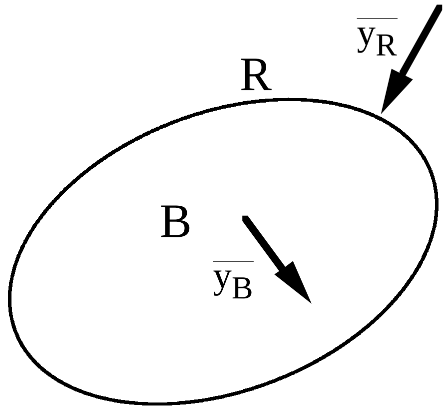

Fig. 1 Control volume B with boundary R

The external generalized forces $\overline{y_B}$ (forces and torques) acting inside the entire volume of the considered reality and the boundary forces $\overline{y_R}$ acting on the boundary of that volume produce a displacement $\overline{z}$ (translations and rotations) of that reality considered as a continuous medium.

In order to understand this behavior in different fields, it is necessary to examine the changes occurring in lines, surfaces and volumes of reference chosen in the control volume. One measure universally accepted for estimating the dynamical change of a multidimensional continuum is the deformation tensor $\overline{\overline{e}}$.

For a deformation map $\overline{\Psi}$ of the continuum, this tensor is equal to half the square of the gradient $\overline{\overline{\Psi}}$ of the deformation map minus the unit tensor $\overline{\overline{I}}$:

$$\overline{\overline{e}} = \frac{1}{2}\left(\overline{\overline{\Psi}}^{T}\overline{\overline{\Psi}}\right) - \overline{\overline{I}} \qquad (2)$$

The internal cohesion forces are described by the stress tensor $\overline{\overline{y}}$.

The internal energy E of the system (neglecting inertial effects) is a scalar resulting from the tensorial double contraction of the deformation tensor by the stress tensor:

$$E = \overline{\overline{y}} : \overline{\overline{e}} \qquad (3)$$

The time rate of change of the internal energy is measured by the tensorial double contraction of the stress tensor by the rate of deformation tensor $\dot{\overline{\overline{e}}}$:

$$\dot{E} = \overline{\overline{y}} : \dot{\overline{\overline{e}}} \qquad (4)$$

Considering that a part of the deformation is irreversible because of energy dissipation, we decompose the deformation into a reversible part $\overline{\overline{e_I}}$ (I standing for Inductance i.e. energy storage) and an irreversible part $\overline{\overline{e_R}}$ (R standing for Resistance i.e. energy dissipation):

$$\dot{E} = \overline{\overline{y}} : \overline{\overline{e_I}} + \overline{\overline{y}} : \dot{\overline{\overline{e_R}}} \qquad (5)$$

3. RELATION TO BOND GRAPH THEORY

It is possible to point out the relation of such a representation of the behavior of a reality to the Bond Graph theory.

3.1 Relation to Bond Graph elements

Considering the energy relations (3), (4) and (5) above, one can find a relationship with the constitutive laws of Bond Graph equations. The reason for this is that, according to the Mobility Analogy (DEL PEDRO 83), $\overline{\overline{e_I}}$ is equivalent to a generalized displacement q, $\dot{\overline{\overline{e_R}}}$ to a generalized flow f, $\overline{\overline{y}}$ to a generalized effort e and the double contraction $\overline{\overline{y}} : \dot{\overline{\overline{e_R}}}$ represents the equivalent to power. This relation allows Tensor Bond Graphs to be used to represent the classical Hook elastic media of Continuum Mechanics:

$$\overline{\overline{y}} = \overline{\overline{\overline{\overline{I}}}} : \overline{\overline{e}} \qquad (6)$$

where $\overline{\overline{\overline{\overline{I}}}}$ is an elasticity fourth order tensor,

and also the Newton viscous media:

$$\overline{\overline{y}} = \overline{\overline{\overline{\overline{R}}}} : \dot{\overline{\overline{e}}} \qquad (7)$$

where $\overline{\overline{\overline{\overline{R}}}}$ is a viscosity tensor.

3.2 Relation to Bond Graph junctions

The Tensor Bond Graph representation used above can be related to the 0 and 1 junctions of Bond Graphs.

In the case of the 1 junction, the global effort is equal to the sum of the efforts of the two basic elements.

The effort on the reversible element I is:

$$\overline{\overline{y_1}} = \overline{\overline{\overline{\overline{I}}}} : \overline{\overline{e}} \qquad (8)$$

The effort on the irreversible element R is:

$$\overline{\overline{y_2}} = \overline{\overline{\overline{\overline{R}}}} : \dot{\overline{\overline{e}}} \qquad (9)$$

The total effort is:

$$\overline{\overline{y}} = \overline{\overline{y_1}} + \overline{\overline{y_2}} \qquad (10)$$

The differential equation of such a continuous medium is:

$$\bar{\bar{y}} = \bar{\bar{\bar{\bar{I}}}} : \bar{\bar{e}} + \bar{\bar{\bar{\bar{R}}}} : \dot{\bar{\bar{e}}} \qquad (11)$$

This equation is that of the Kelvin medium of Continuum Mechanics. For such a medium, a sudden application of a stress produces no immediate deformation because the irreversible part of the medium does not react instantaneously. Instead a deformation will gradually build up as the reversible part takes a greater and greater share of the stress. When the stress is removed, the deformation disappears gradually.

In the case of 0 junctions, the global displacement is equal to the sum of the displacements of the two connected elements:

$$\bar{\bar{e}} = \bar{\bar{e}}_1 + \bar{\bar{e}}_2 \qquad (12)$$

The displacement on the reversible element is:

$$\bar{\bar{y}} = \bar{\bar{\bar{\bar{I}}}} : \bar{\bar{e}}_1 \qquad (13)$$

and the flow on the second element is:

$$\bar{\bar{y}} = \bar{\bar{\bar{\bar{R}}}} : \dot{\bar{\bar{e}}}_2 \qquad (14)$$

which gives as differential equation for the 0 combination of the two elements:

$$\frac{\bar{\bar{\bar{\bar{R}}}}}{\bar{\bar{\bar{\bar{I}}}}} : \dot{\bar{\bar{y}}} + \bar{\bar{y}} = \bar{\bar{\bar{\bar{R}}}} : \dot{\bar{\bar{e}}} \qquad (15)$$

or

$$\dot{\bar{\bar{e}}} = \frac{1}{\bar{\bar{\bar{\bar{R}}}}} : \bar{\bar{y}} + \frac{1}{\bar{\bar{\bar{\bar{I}}}}} : \dot{\bar{\bar{y}}} \qquad (16)$$

This equation represents the classical Maxwell medium of Continuum Mechanics. A sudden application of stress on such a medium induces an immediate deflection by the reversible part followed by a creep due to the irreversible part. On the other hand, a sudden deformation produces an immediate stress due to the reversible part, followed by a stress relaxation due to the irreversible part.

4. TENSOR BOND GRAPHS

The Bond Graphs effort and flow vectors may represent the reality considered. For example, the deformation of a medium in a three-dimensional space $x_{i,\ i = 1..3}$ can be represented by a row vector of six deformation components, three translations and three rotations defined as follows:

$$\bar{e}^T = \left[\varepsilon_{x1x1}, \varepsilon_{x2x2}, \varepsilon_{x3x3}, \gamma_{x1x2}, \gamma_{x2x3}, \gamma_{x3x1}\right]$$

where the $\varepsilon_{x_i x_i}$ are the translations and the $\gamma_{x_i x_j}$ are the rotations. This is like a vector of displacement {q} (translations and rotations) in the Bond Graph sense defined as:

$$\bar{q}^T = \left[q_{x_1 x_1}, q_{x2x2}, q_{x3x3}, q_{x1x2}, q_{x2x3}, q_{x3x1}\right]$$

We can start with the Kelvin medium and put its deformation in vectorial form resulting in the matrix differential equation:

$$\bar{y} = \bar{\bar{I}}\,\bar{e} + \bar{\bar{R}}\,\dot{\bar{e}} \qquad (17)$$

which corresponds to the following Bond Graph notation:

$$\{e\} = [I]\{q\} + [R]\{f\} \qquad (18)$$

where [I] is a stiffness matrix, [R] is a damping matrix, {e} is the effort, {q} the displacement and {f} the flow. This is the equation of a Tensor Bond Graph for a three-dimensional space with vectors of efforts and flows arranged in a summation of efforts corresponding to a 1 junction as shown in Fig. 2:

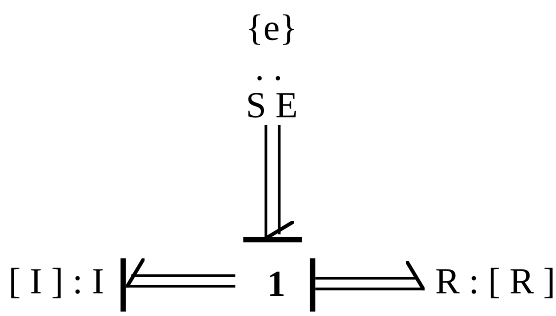

Fig. 2 Continuous media Tensor Bond Graph

When we consider non-physical systems, the Tensor Bond Graph is composed of Pseudo Bonds because of the units involved. Tensor Pseudo Bond Graphs are used when the analysis extends to social and biological systems.

The three-dimensional space Tensor Bond Graph above is composed of a multiport I field and a multiport R field, which contain the coefficient matrices. The matrices representing the behavior of the system are obtained utilizing the ability of the Camp-G software (GRANDA 95) to process multiport fields. Such a Bond Graph entered in Camp-G is shown in Fig. 3 hereunder:

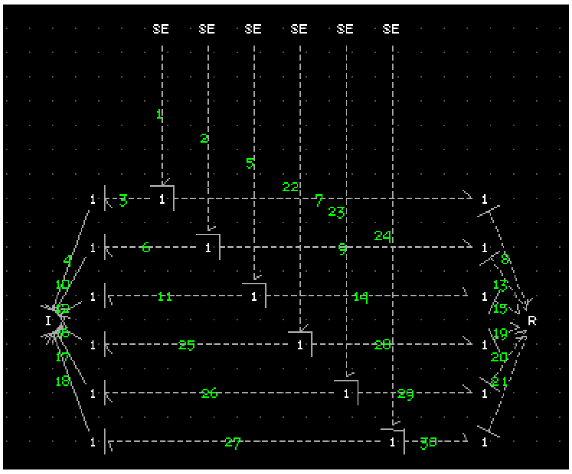

Fig. 3 Continuous media three-dimensional space model

The $\left[\mathrm{I}\right]$ matrix containing the stiffness coefficients of such a system is shown in Fig. 4:

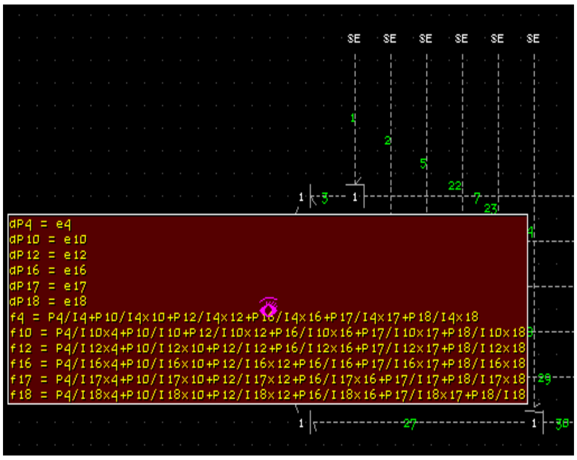

Fig. 4 [I] Multiport coefficient matrix

This 6x6 matrix I is typical of an anisotropic medium whose properties are varying in space. Among the 36 coefficients of this matrix, only 21 are independent because of the symmetry of the angular deformations. In this general case, 21 independent measurements are mandatory in order to identify the 21 coefficients. In case there are symmetries in the behavior of the medium, the number of independent coefficients goes down to 9 for orthotropic media and down to 2 for isotropic media which are symmetric with respect to every plane and every axis.

The $\left[\mathrm{R}\right]$ matrice is shown in Fig. 5:

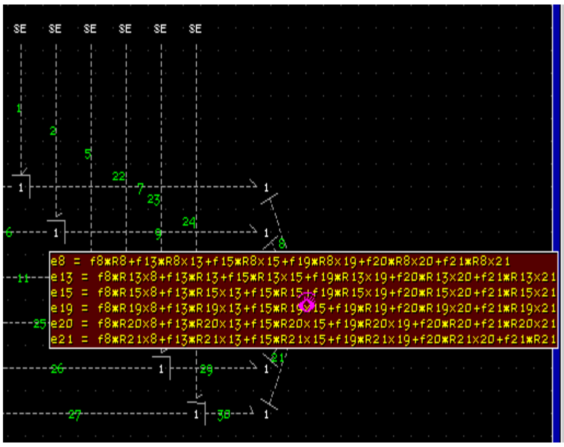

Fig. 5 $\left[\mathrm{R}\right]$ Multiport coefficient matrix

In the case of the Maxwell model, we can also use the matrix differential equation:

$$\frac{\bar{\bar{R}}}{\bar{\bar{I}}}\,\dot{\bar{y}} + \bar{y} = \bar{\bar{R}}\,\dot{\bar{e}} \qquad (19)$$

giving in turn the following Bond Graph equation:

$$\frac{\left[\mathrm{R}\right]}{\left[\mathrm{I}\right]}\,\{\dot{e}\} + \{e\} = \left[\mathrm{R}\right]\,\{f\} \qquad (20)$$

Multiplying both sides by the inverse of $\left[\mathrm{R}\right]$ we get:

$$\left[\frac{1}{\mathrm{I}}\right]\{\dot{e}\} + \frac{\{e\}}{\left[\mathrm{R}\right]} = \{f\} \qquad (21)$$

which is obviously the sum of two flows equal to $\{f\}$. Such representation in Bond Graph form is only possible by summing the flows using a 0 junction. Therefore, the three-dimensional space Tensor Bond Graph takes the form shown in fig. 6 below:

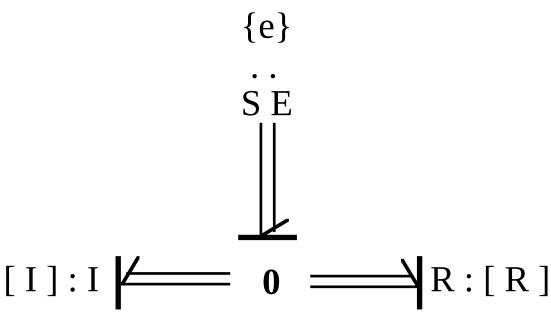

Fig. 6 Bond Graph model of a Maxwell viscoelastic medium

5. TYPICAL BEHAVIOURS

We have seen that the Bond Graph parameter matrices represent the behavior of the medium considered. A number of viscoelastic media models can be constructed easily with the basic Tensor Bond Graphs presented above (CHUNG 96). The possibility of introducing non-linear behaviors with Bond Graphs (GRANDA 99) allows all the spectra of rheological viscoelastoplatic media models to be represented (COUARRAZE 00), including hysteresis due to sensibility of the medium to the application of various stress frequencies (VERHAS 97).

6. EXAMPLE IN ECONOMICS

We are now in a position to follow up work done in Bond Graph representation of social systems (BREWER 76, BREWER 77, BREWER 82) as well as biological systems (BREWER 80, SCHNAKENBERG 81, BREWER 91). This can be done according to the general manner of classifying the dynamical variables into efforts and flows, as defined in the general scientific field (JONES 71, PETERSON 79, HEZEMANS 86).

Let us take a very simple example in the field of economics. Consider a banker who wants to understand the dynamics of a financial market. For that purpose, he uses two discriminant criteria (stocks and bonds, for instance) in order to modelize the flow of investors moving on that market.

We see immediately that the banker is facing a two-dimensional space problem. Using the Computer Aided Modeling Program Camp-G gives the Tensor Pseudo Bond Graph shown in fig. 7, with a three-components displacement vector (two translations and one rotation) and a three-components effort vector representing the market motivations driving investors and are like the effort variable of the Bond Graphs.

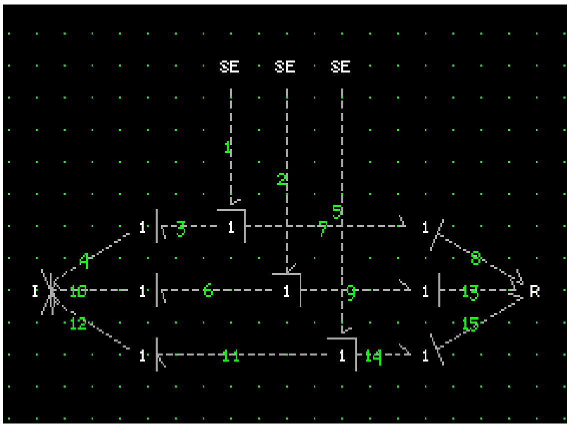

Fig. 7 Financial Tensor
Pseudo Bond Graph in Camp-G

The elasticity matrix of the market produced with Camp-G is shown in Fig. 8:

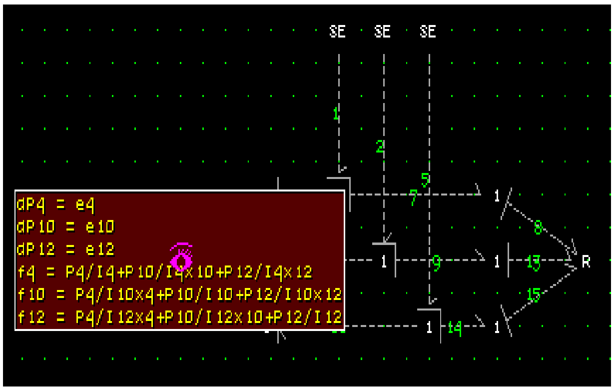

Fig. 8 Elasticity matrix of the market

This matrix shows the equation of the multiport element on the left. It allows the banker to realize that in order to understand the dynamics of the market considered, he needs to identify these coefficients in the DataBase containing the information about the market.

The fluidity matrix of the market, which can be compared to the multiple R matrix of Bond Graphs, allows the banker to understand some irreversible behaviors of the investors and is shown in Fig. 9:

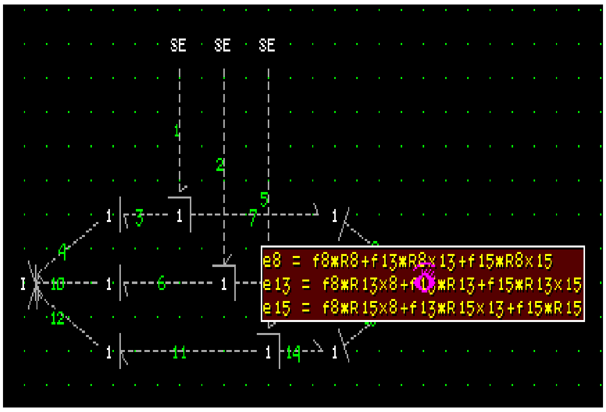

Fig. 9 Fluidity matrix of the market

It is interesting to note that the knowledge the banker gets at this point allows him to perform a Dimensional Analysis of the variables and parameters of the model, ensuring that his representation of the reality is coherent (SZIRTES 98).

Furthermore, the Camp-G program, thanks to its interfaces to Matlab and Simulink, produces a computable model of the market. The variables, parameters and differential equations constituting this model are shown in Fig.10 below:

```
  function p_qdot = campgequ(t,p_q)
%  ...........campgequ.m   CAMP-G/MATLAB function ...........
%  System differential equations, state Vectors
%
  global I4 I4x10 I4x12 R8 R8x13 R8x15 I10x4 I10 I10x12 I12x4  ...
  I12x10 I12 R13x8 R13 R13x15 R15x8 R15x13 R15
  global SE1 SE2 SE5
  global TIME STEP EFFORTS FLOWS
% System Differential Equations-First Order Form
%...... Define State Variables ......
        P10= p_q(1) ;           P12= p_q(2) ;
        P4= p_q(3) ;
% p_q = [P10; P12; P4] ;
%...... Define derivatives (dp,dq) and output variables (e,f) ......
        e1=SE1 ;                      e2=SE2 ;
        f4=P4/I4+P10/I4x10+P12/I4x12 ;
        e5=SE5 ;                      f10=P4/I10x4+P10/I10+P12/I10x12 ;
        f12=P4/I12x4+P10/I12x10+P12/I12 ;
        f3=f4 ;                       f6=f10 ;
        f11=f12 ;                     f7=f3 ;
        f8=f7 ;                       f9=f6 ;
        f14=f11 ;                     f1=f3 ;
        f2=f6 ;                       f5=f11 ;
        f13=f9 ;                      f15=f14 ;
        e13=f8*R13x8+f13*R13+f15*R13x15 ;
        e15=f8*R15x8+f13*R15x13+f15*R15 ;
        e9=e13 ;                      e8=f8*R8+f13*R8x13+f15*R8x15 ;
        e14=e15 ;                     e7=e8 ;
        e6=e2-e9 ;                    e10=e6 ;
        e11=e5-e14 ;                  e12=e11 ;
      dP10=e10 ;                    dP12=e12 ;
        e3=e1-e7 ;                    e4=e3 ;
      dP4=e4 ;
```

Fig. 10 The Matlab file containing the computable model of the market

The use of the Matlab files provided by Camp-G allows the banker to generate automatically the State Space Form of the market model. This State Space Form is shown in Fig. 11:

```
Inputs vector
  u=[  SE1 SE2 SE5 ]
State variables vector
 p_q=[P10;P12;P4];
 A  MATRIX
    [  R13x8    R13    R13x15      R13x8    R13      R13x15
    [- ----- - --- - ------ , - ----- - ------ - ------ ,
    [  I4x10    I10    I12x10      I4x12    I10x12   I12

    R13x8    R13     R13x15]
  - ----- - ----- - ------]
    I4      I10x4   I12x4  ]

    [  R15x8    R15x13    R15         R15x8    R15x13    R15
    [- ----- - ------ - ------ , - ----- - ------ - --- ,
    [  I4x10    I10       I12x10      I4x12    I10x12    I12

    R15x8    R15x13    R15  ]
  - ----- - ------ - -----]
    I4       I10x4     I12x4]

    [   R8      R8x13    R8x15        R8      R8x13     R8x15
    [- ----- - ----- - ------ , - ----- - ------ - ----- ,
    [  I4x10    I10      I12x10       I4x12   I10x12    I12

     R8     R8x13    R8x15]
  - ---- - ----- - -----]
    I4      I10x4    I12x4]
 B  MATRIX
                                        [0     1     0]
                                        [             ]
                                        [0     0     1]
                                        [             ]
                                        [1     0     0]
 C  MATRIX
                                      [sb    sb    sb]
 D  MATRIX
                                      [sb    sb    sb]
 System Order =  3
```

Fig. 11 The State Space Form of the market model

With this State Space Form, many tools developed for Automatic Control in the Engineering field, like Matlab toolboxes, Simulink or Sysquake, become available for analysis of financial markets.

Our objective, which was to obtain a representation allowing the use of automatic control tools in the field of Social and Life Sciences, is fulfilled.

7. CONCLUSION

In Social and Life Science fields, the scientific understanding of a situation necessitates a combination of scalar, vectorial and tensorial variables and parameters into a coherent calculus scheme. Since we have established a relationship of such a scheme with Bond Graphs, then the constitutive laws of Bond Graph theories allow the calculation of the state and anticipated evolution in time of systems considered in the fields of social and life sciences.

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